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Technical Note

Dual free-convective flows in a horizontal annulus with a constant heat flux wall

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Abstract

Natural convection in a horizontal annulus with a constant heat flux wall is investigated for the fluids of $0.2 \leqslant Pr \leqslant 1$. The outer cylinder is kept at a constant temperature, and the inner cylinder is heated with a constant heat flux. By using a numerical approach in solving the unsteady governing equations of flow and temperature fields, it is shown that dual steady solutions exist above a critical Rayleigh number.

1. Introduction

Though the natural convection in a horizontal annulus kept at constant wall temperature difference has been studied extensively, relatively few studied the existence of multiple solutions and bifurcation phenomena. Most research has focused the main attention on the rate of heat transfer at the walls. However, a horizontal annulus with a heated inner cylinder is an interesting physical system where thermal and hydrodynamic instabilities coexist. Yoo [1] showed that the two instabilities could induce diverse fluid flows and transition phenomena in a narrow-gap annulus. And it was found that two kinds of flow patterns were realized in a wide-gap annulus [2]. Mizushima et al. [3] performed a bifurcation analysis to investigate the stability of flows, and confirmed the existence of dual stable steady solutions for a fluid with $Pr = 0.7$ in the annuli with several aspect ratios [4].

In this study, we investigate multiple solutions in an annulus with a constant heat flux wall. The heat flux condition on the wall can describe the direct electrical heating of cylinder by Joule effect, and so it is important from the viewpoint of experiment [5,6] and application. Although some authors [5–8] investigated the natural convection in the annulus with constant heat flux walls, no one found multiple solutions. The fluid is contained between two horizontal concentric circular cylinders, where the surface of the inner cylinder is maintained at a constant heat flux (q_H), and the outer cylinder is kept at a constant temperature (T_0): Fig. 1(a). Dual steady flows are found, and the characteristics of the flows are investigated.

2. Analysis

The problem configuration and coordinate system are shown in Fig. 1(a). The 2-D dimensionless governing equations under Boussinesq approximation and the boundary conditions can be written as follows [2,8]:

$$\frac{\partial \omega}{\partial t} = J(\Psi, \omega) + Pr\nabla^2\omega - Pr\,Ra \times \left[\sin(\phi)\frac{\partial \theta}{\partial r} + \cos(\phi)\frac{\partial \theta}{r\partial \phi}\right] \quad (1)$$

$$\omega = -\nabla^2\Psi \quad (2)$$

$$\frac{\partial \theta}{\partial t} = J(\Psi, \theta) + \nabla^2\theta \quad (3)$$

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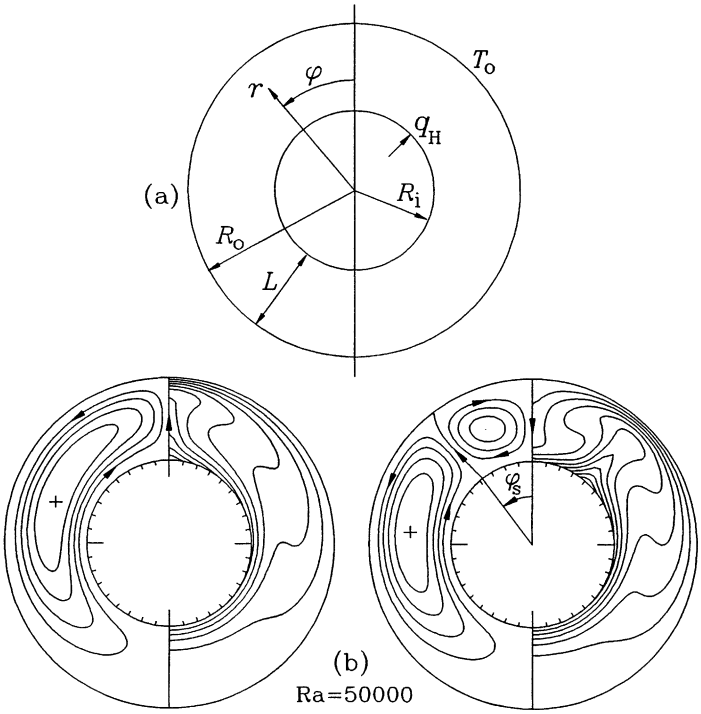

Fig. 1. Problem configuration (a), and the streamlines and isotherms of dual solutions for $Pr = 0.7$ and $Ra = 5 \times 10^4$ (b). In (b), the left and right represent 'upward' and 'downward' flows, respectively. In the downward flow, the angle ϕ_S represents the location of separation point between two cells on the wall.

$$\Psi = \frac{\partial \Psi}{\partial r} = 0, \quad \omega = -\frac{\partial^2 \Psi}{\partial r^2}, \quad \frac{\partial \theta}{\partial r} = -1, \quad \text{at } r = r_i \tag{4}$$

$$\Psi = \frac{\partial \Psi}{\partial r} = 0, \quad \omega = -\frac{\partial^2 \Psi}{\partial r^2}, \quad \theta = 0, \quad \text{at } r = r_0 \tag{5}$$

where Ψ, ω, and θ are the dimensionless streamfunction, vorticity, and temperature, respectively.

The Rayleigh number (Ra) and Prandtl number (Pr) are defined as $Ra = \alpha g(q_H L/k)L^3/\kappa\nu$ and $Pr = \nu/\kappa$, respectively, where k, α, κ, and ν are thermal conductivity, thermal expansion coefficient, thermal diffusivity, and kinematic viscosity of the fluid, respectively, L is the gap width of annulus, and g is the acceleration due to gravity. The dimensionless temperature (θ) is defined as $\theta = k(T - T_0)/q_H L$. The flow is assumed symmetric about the vertical plane through the center of cylinders.

We define the average heat transfer coefficient ($\bar{h}$) with the mean temperature ($T_{m,i}$) of the inner cylinder [8] as

$$\bar{h} = \frac{q_H}{(T_{m,i} - T_0)} \tag{6}$$

The mean Nusselt number ($\overline{Nu}$) can be defined as

$$\overline{Nu} = \frac{\bar{h}L}{k} = \frac{1}{\theta_{m,i}} \tag{7}$$

and the local heat flux on the outer cylinder (q_0) is given by

$$\frac{q_0}{q_H} = -\frac{\partial \theta}{\partial r} \quad \text{at } r = r_0 \tag{8}$$

The governing equations (1)–(5) are solved numerically by the finite difference method [1]. This study considers an annulus with $R_0/R_i = 2$ in the range of $0.2 \leqslant Pr \leqslant 1$ and $Ra \leqslant 2 \times 10^5$, and uses a ($r \times \phi$) mesh of (45 × 65); the mesh has been confirmed to give sufficiently accurate results from the mesh test.

3. Results and discussion

This study investigates the two types of flows distinguished by the flow direction atop the annulus. We will name the flow where the fluid near the top of annulus ($\phi = 0$) ascends as 'upward flow', and descends as 'downward flow' [2]. When $Pr = 0.7$, the upward flow is obtained starting from zero initial condition($\omega = \theta = 0$), and the downward flow can be also obtained when Ra is sufficiently large by imposing a numerical disturbance during a short initial period. An example of the dual flows for $Pr = 0.7$ is presented in Fig. 1(b) with $Ra = 5 \times 10^4$. In the downward flow, we can see two counter-rotating eddies in a half annulus, and define ϕ_S as the angle representing the location of separation point between the two cells on the wall (Fig. 1(b)). The approximate size of the cell on the top of annulus can be measured from the values of ϕ_S's on inner ($\phi_{S,i}$) and outer ($\phi_{S,o}$) cylinders.

The temperature distribution on inner cylinder and the local heat flux distribution on outer cylinder of the dual solutions are shown in Fig. 2. When $Pr = 0.7$, dual solutions exist at $Ra \geqslant 5700$, but only upward flow is found at $Ra \leqslant 5600$. The distributions for upward and downward flows at the top region of annulus ($\phi < 90°$) are significantly different each other. But there is no great difference of temperature distribution on the inner cylinder at the bottom region ($\phi > 90°$) . In the upward flow, both the maximum temperature on inner cylinder and the maximum heat flux on outer cylinder always occur at the uppermost point of annulus ($\phi = 0$). In the downward flow, however, the locations of the points are shifted with Ra by the variation of eddy shape.

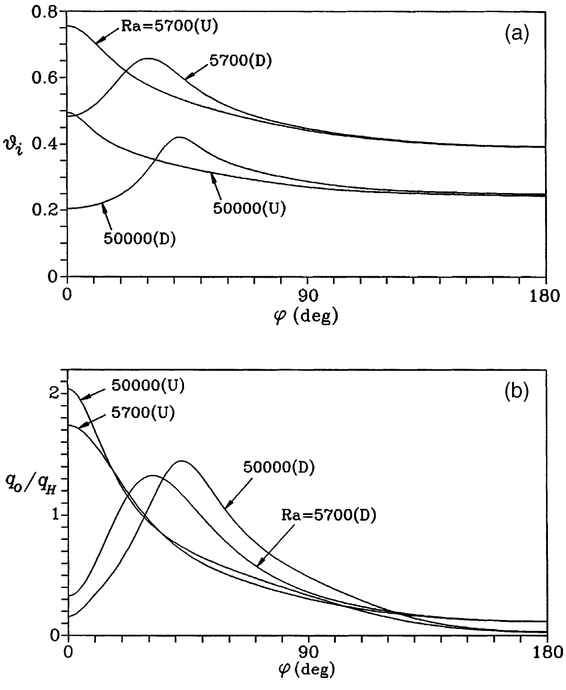

Fig. 2. Distributions of temperature and heat flux on the walls for the dual solutions of $Pr = 0.7$ with $Ra = 5700$ and 5×10^4: (a) local temperature distribution on inner cylinder [$\theta_i(\phi)$]; (b) local heat flux distribution on outer cylinder [$q_0(\phi)/q_H$]. The letters 'U' and 'D' denote the 'upward' and 'downward' flows, respectively.

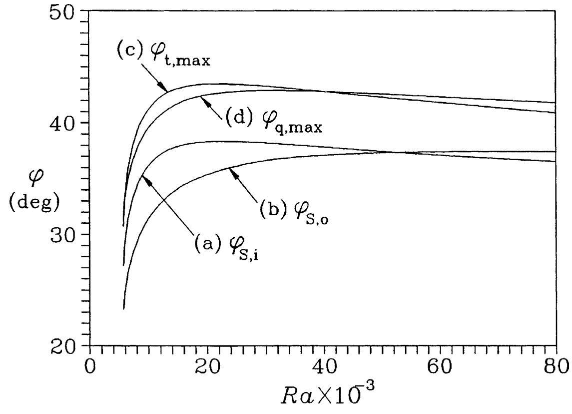

Fig. 3. Angles indicating the locations of separation points, points of maximum temperature and heat flux on the walls, in the downward flow of $Pr = 0.7$: (a) separation point on inner cylinder ($\phi_{S,i}$); (b) separation point on outer cylinder ($\phi_{S,o}$); (c) point of maximum temperature ($\phi_{t,max}$) on inner cylinder; (d) point of maximum heat flux ($\phi_{q,max}$) on outer cylinder. All the angles are measured from the top of annulus (Fig. 1(a)).

Fig. 3 shows the angles indicating the locations of separation points ($\phi_{S,i}$, $\phi_{S,o}$), points of maximum temperature ($\phi_{t,max}$) and heat flux ($\phi_{q,max}$) on the walls, in the downward flow of $Pr = 0.7$. At the inner cylinder, the point of maximum temperature always locates below the separation point of two cells ($\phi_{t,max} > \phi_{S,i}$). And $\phi_{t,max}$ and $\phi_{S,i}$ have nearly identical behavior with respect to Ra. The eddy on the top of annulus does not continuously grow with increase of Ra, and $\phi_{S,i}$ has its maximum value at $Ra \approx 2.5 \times 10^4$. The angles, $\phi_{t,max}$ and $\phi_{q,max}$, at the same Ra have nearly identical values within 5% difference.

The mean Nusselt numbers of $Pr = 0.7$ as functions of Ra are presented in Fig. 4. The bifurcation phenomenon

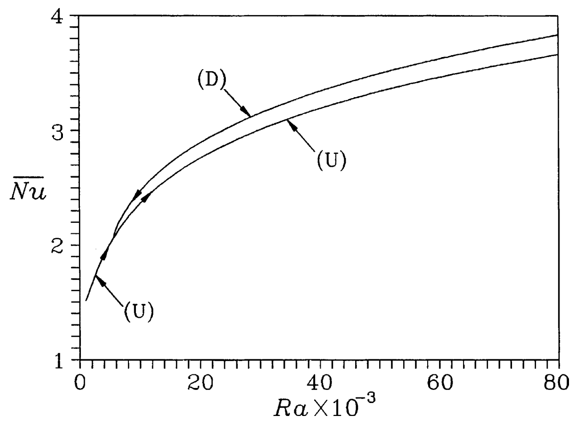

Fig. 4. Mean Nusselt number ($\overline{Nu}$) as a function of Ra for $Pr = 0.7$. The letters 'U' and 'D' denote the 'upward' and 'downward' flows, respectively.

is also shown in Fig. 4 with arrows on the Nusselt number curves. The transition phenomenon is investigated by the following manner. Once having obtained a downward flow at a high Ra ($Ra = 8 \times 10^4$), Ra is decreased and the solution is found by letting the initial condition be the previously obtained downward flow. It is found that at $Ra < Ra_c$ (a critical Ra), an upward flow is realized although the initial condition is downward flow. And starting from an upward flow of a small Ra ($Ra = 1000$), the solutions for the higher Ra are obtained with increase of Ra. Fig. 4 shows that both upward and downward flows exist at $Ra > Ra_c$, and $\overline{Nu}$ of downward flow is greater than that of upward flow. As Ra is decreased, a transition from downward to upward flow occurs. But the transition from upward to downward flow does not occur.

When $0.3 \leqslant Pr \leqslant 1$, downward flow has not been obtained from zero initial condition if we do not disturb the flow numerically. At $Pr = 0.2$, however, the downward flow can be obtained naturally from the transient development of flows after impulsive imposing of heat flux on the inner cylinder. The transient development to a downward flow at $Pr = 0.2$ is shown in Fig. 5 with $Ra = 5 \times 10^4$. After heating the inner cylinder, an upward flow with a crescent-shaped eddy appears first (Fig. 5(a)). After a short period of time, however, the boundary layer on the inner cylinder is separated from the wall at a point other than the uppermost point of the inner cylinder ($\phi = 0$), and a small counter-rotating eddy attached on the inner cylinder is formed (Fig. 5(b)). Once having created a counter-rotating eddy, the eddy grows in size and strength (Fig. 5(c)–(f)). And finally, a steady downward flow having approximately square-shaped cell on the top region of annulus is established. The global behavior of the transient development to downward flow is similar to that of the case with isothermal walls [2]. When the surface of inner wall is subjected to a constant heat flux condition, however, a new small eddy attached to the top region of the inner cylinder appears during a short period of the transient process. Fig. 5(c)–(f) shows the creation of a small eddy on the top of inner cylinder (Fig. 5(c) and (d)) and coalescence of the eddy with a large one (Fig. 5(e) and (f)).

When the annulus has isothermal walls [2], downward flow can be obtained naturally by the impulsive heating of inner cylinder at $0.3 \leqslant Pr \leqslant 0.5$, and hysteresis phenomenon occurs at $0.3 \leqslant Pr \leqslant 0.4$. For an annulus with a constant heat flux wall, however, the flow can not be obtained without numerical disturbances and the hysteresis phenomenon is not observed, at $0.3 \leqslant Pr \leqslant 1$.

Finally, Fig. 6 presents the boundary of dual solutions on the Pr–Ra plane in the range of $0.2 \leqslant Pr \leqslant 1$. The curve of Ra_c lies between the two boundary curves. It is observed that Ra_c as a function of Pr has its minimum value at about $Pr \approx 0.25$: as Pr increases, Ra_c is decreased at $0.2 \leqslant Pr < 0.25$, but is increased at $0.25 < Pr \leqslant 1$. When the inner and outer cylinders are held at different uniform temperatures [2], the critical Rayleigh number is continuously decreased with increase of Pr at $0.3 \leqslant Pr \leqslant 1$. In the case of the isothermal walls [2], however, the Rayleigh number is defined with the temperature difference between two walls, and so it may not possible to compare the critical Rayleigh curves directly. Downward flow exists by the combined effects

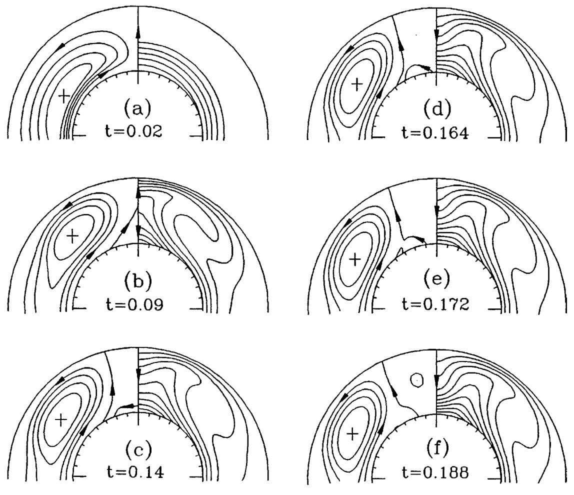

Fig. 5. Transient development of flows to a downward flow when $Pr = 0.2$ and $Ra = 5 \times 10^4$: at (a) $t = 0.02$; (b) $t = 0.09$; (c) $t = 0.14$; (d) $t = 0.164$; (e) $t = 0.172$; (f) $t = 0.188$. The initial conditions are $\omega = \theta = 0$, and the inner cylinder is suddenly heated with a constant heat flux.

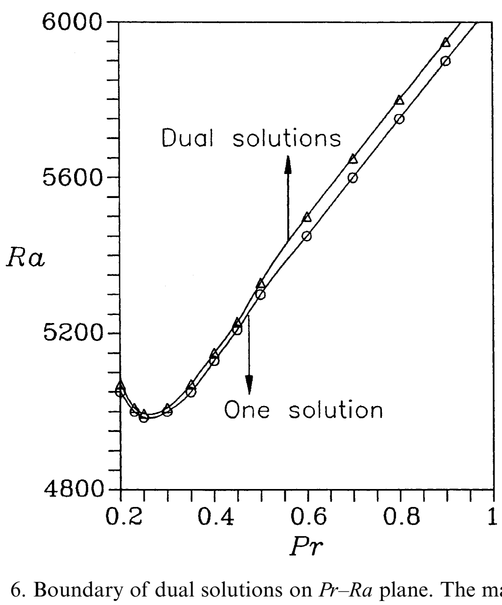

Fig. 6. Boundary of dual solutions on Pr–Ra plane. The marks '△' and 'O' on the curves indicate the calculation points at which dual solutions and one solution are found, respectively.

of thermal and hydrodynamic instabilities which are dependent on *Pr*. And the strength of the thermal instability on the top of annulus is dependent on the temperature difference between two walls. It seems that the non-uniform temperature distribution on the inner cylinder heated with a constant heat flux makes the behavior of Ra_c with respect to *Pr* somewhat complex.

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